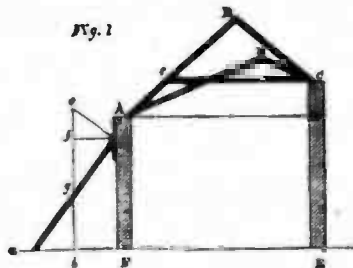


FA, EC the heights of the back and front walls, and AB or AH the longer rafter which is given; then are BA, BC, or HA, HC, the positions of the rafters, the roof admitting of two pitches as we shall see anon; and since neither of them is given by the question, it becomes a matter of inquiry to determine the nature of the roof itself, and also the effect of its thrust upon the walls.

Let FABCE represent a vertical transverse section of the roof and walls, aP one of the struts, P the point in the wall at which it is applied, and g the position of its centre of gravity. At P , the upper ex-



tremity of the strut which presses on the wall, erect the perpendicular Pc , to meet the vertical line bc , passing through the centre of gravity in e , and let fP be perpendicular to bc .

The triangles PaF and geP are similar, and by the principles of mechanics, the pressure of the strut aP in the direction eP , perpendicular to its surface, is as eP , and may, therefore, be represented by it; hence it follows, that the direct pressure is also as fP , and may consequently be represented thereby. Now, in estimating the force of the strut upon the wall in the direction of its length, we may assimilate that force to a weight applied at the centre of gravity, and acting in a direction perpendicular to the horizon; let eg denote this weight, then by the property of similar triangles, we have

$$aP : aF :: ge : eP = \frac{eP \times ge}{aP}$$

but the triangles PaF and PeF are also similar; hence it is

$$aP : PF :: eP : fP = \frac{PF \times eP}{aP}$$

Now this is the force with which the strut aP resists the wall in the direction fP , and if the value of the force in the direction eP be substituted, we obtain

$$fP = \frac{aF \times PF \times ge}{aP^2}$$

The equation just obtained expresses the value of the force which resists the deflection of the wall in the direction fP , and the leverage by which it gains its power is PF ; hence, the momentum or mechanical effect in resisting the wall, is

$$\text{MOMENTUM} = \frac{aF \times PF^2 \times ge}{aP^3}$$

In this equation the unknown quantity is PF , and when that becomes known, the other quantities aF and ge are also known, but by the conditions of the question, the momentum or mechanical effect of the strut is to be a maximum; consequently, by equating the fluxion of the above expression with zero and transposing the terms, we get

$$3PF^2 = 2aP^2, \text{ and this by reduction, gives } PF = \frac{aP}{\sqrt{6}} = 0.408248P$$

Hence it appears, that if the length of the strut in any case be multiplied by the decimal 0.408248, it will give the height of the point in the wall, at which it is to be applied to produce the greatest effect.

In the present case, the length of the strut is 12½ feet; therefore, by multiplication, it is $PF = 12.5 \times 0.408248 = 5.1031$ feet nearly, and the distance from the bottom of the wall where the foot of the strut is placed, is $Fa = 7.22$ feet nearly. But if the point in the height of the wall be previously fixed on, and the length of the strut not known, this length may readily be found in all cases, by making with the wall at the point of application an angle equal to $35^\circ 16'$; or by Plane Trigonometry we have

$$aP : \text{radius} :: PF : \cos aPF; \text{ that is, } 12.5 : 1 :: 10.21 : 0.81650 = \text{nat. cos. } 35^\circ 16';$$

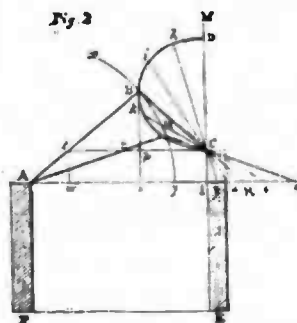
and this angle is constant in the case of maximum resistance, for every length of strut that does not reach above the summit of the wall.

This is the method of resolving the problem, in so far as it respects the position of the strut, and in the next place, we have to determine the length of the shorter rafter, or the pitch of the roof, and from this to assign its effects in pushing out the wall from the perpendicular position.

In order to this, let AI , fig. 2, be the span of the roof; AF and CE the back and front walls, the one being higher than the other by a quantity equal to IC , and AB or AH the length of the longer rafter, which is supposed to be known. Produce IC directly upwards to M , and set off ID equal to AB or AH , the length of the given rafter; in AI produced beyond I , take any number of points whatever, such as at 1, 2, 3, 4, the more the better for an accurate construction. Then, through C , the point where the rafter meets the top of the higher wall, and the several assumed points, draw the straight lines $1h$, $2i$, $3k$, $4l$, and on these lines respectively set off the several distances equal to the given rafter AB , or vertical line ID ; then are D , h , i , B , k , l , C , points in the curve described by the point D ; while the lower end of ID is drawn along the horizontal line 14 , the line itself passing always through the point C . Through the several points just named let the curve $DhBkHC$ be traced, and we shall have a curve of the fourth order, in which the summit or ridge of the roof occurs. On the point A as a centre, with a radius equal to the length of the given rafter, describe the circular arc $xBIHy$, intersecting the curve in the points B and H , and draw AB , AH , and CB , CH ; then will ABC and AHC be the pitches of the roof required. If the circular arc $xBIHy$ does not intersect the curve, but merely touches it in a single point, the roof will only admit of one pitch, having its ridge in the point of contact; and if the circular arc does not touch the curve at all, the construction is impossible with the given length of rafter.

Such, therefore, is the method of delineation under the conditions specified, and if CB and CH be applied to the scale of construction, the length of the shorter rafter for both pitches will be approximately known, and this in most practical cases will be sufficient, and beyond this point mechanics in general will not attempt to go; but in order to render the solution of the problem more complete, we think proper to shew the mode of calculation also.

Through the point C , draw the straight line Ct parallel to the horizon, and meeting AB in



the point t , and AH in z , and from the points t and B let fall the perpendiculars tw and Bw ; then are the triangles Atw , ABt , and tBw similar among themselves. Therefore, by the property of similar triangles, we have

$$At : Atw :: Bt : tw = \frac{Atw \times Bt}{At}$$

$$\text{but } tw = 2pw = \frac{2Atw \times Bt}{At}$$

and by addition, we get $At + tw = AI$; that is

$$At^2 - 52At + 411At^2 + 468At = 6084$$

Here we have an equation of the fourth degree with all its terms present, consisting of three positive roots and one negative. In this equation, At is the unknown quantity, and its

* It may be proper to remark, that the several coefficients in this equation, as well as the absolute number 6084, are found by introducing the dimensions of the building as specified in the question.

two values will be expressed by two of the positive roots of the equation; which roots by approximation are found to be

$$At = 4.4 \text{ feet, and } Az = 9.1 \text{ feet very nearly.}$$

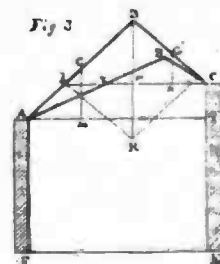
By repeating the steps of the approximation the values of At and Az may be found to a higher degree of accuracy; but it is unnecessary in the present instance, as the numbers found are sufficiently accurate for every practical purpose. Therefore, by subtracting each of these values from 13, the length of the given rafter, we get

$$BC = Bt = 13 - 4.4 = 8.6 \text{ feet; and } HC = Hz = 13 - 9.1 = 3.9 \text{ feet;}$$

so that for the higher pitch ABC , the rafters are 13 and 8.6 feet; and for the lower pitch AHC , they are 13 and 3.9 feet.

Having in the previous part of our solution determined the position of the strut, so that it shall present the greatest possible horizontal resistance to the inclination of the wall; and having, moreover, shewn the method of finding the length of the shorter rafter when that of the longer one is given, so that both sides of the roof may have the same inclination, or that a plummet suspended from the ridge may bisect the angle of concourse; it now remains for us to inquire, if the walls were of the proper dimensions for resisting the thrust of the roof, on the supposition that the rafters are free, and totally unrestrained by ties of any kind.

For this purpose, let ABC and AHC , fig. 3, be the two positions or pitches that we have



shewn the roof susceptible of assuming, and let G and G' be the positions of the centres of gravity of the sides of the roof respectively; through the centres just named let the vertical lines Gm and $G'n$ be drawn, and they will represent the directions in which each side of the roof acts, considering the whole weight of each side to be concentrated in these points. Now it is very obvious, from the principles of mechanics, that the weight of each side acting in the directions Gm and $G'n$, will produce the same effect upon the walls as if the whole weight of the roof were applied at the point B , and acting in the direction of gravity BR . Let BR denote the whole weight of the roof, and upon BR , as a diagonal, construct the parallelogram $BCRA$; then is BR resolved into its component forces BC and BA , acting in the directions of the rafters BC and BA ; but BC is equal to Bh , as can easily be shewn by the principles of geometry and the nature of the figure; hence it follows, that the thrust on the walls is precisely the same in the direction of each rafter; it is, however, the horizontal thrust that enters into the consideration of the case in question, and to obtain the magnitude of that thrust, we must resolve the oblique force Bh or BC , which is equal to it, into the co-ordinate or component forces Br and Ar , the one perpendicular, and the other parallel to the horizon. Now Br , the force which acts in the direction of gravity, has no effect whatever in thrusting out the wall from its vertical position; it is only Ar , or that which acts in a direction parallel to the horizon that becomes effective, and we straightway proceed to determine its magnitude.

According to the data of the question, the length of the building is 34 feet, and the length of the longer rafter is 13 feet, and by a calculation deduced from the principles of construction, we have found the length of the shorter rafter BC , to be $13 - 4.4 = 8.6$ feet; but it is moreover stated, that the weight of the tiles and the timber give an average weight of fifteen pounds upon a square foot of surface. Now the whole surface for the upper pitch ABC , is $(13 + 8.6) \times 34 = 21.6 \times 34 = 734.6$ square feet, and this being multiplied by fifteen